Racionalización de denominadores
Casos: raíz cuadrada, raíces de índice mayor que dos y racionalización de un binomio mediante el conjugado.

1. Raíz cuadrada en el denominador

a) \( \frac{3}{\sqrt{2}} = \frac{3\sqrt{2}}{\sqrt{2}\sqrt{2}} = \frac{3\sqrt{2}}{2} \)

b) \( \frac{2}{\sqrt{3}} = \frac{2\sqrt{3}}{\sqrt{3}\sqrt{3}} = \frac{2\sqrt{3}}{3} \)

c) \( \frac{\sqrt{2}}{\sqrt{3}} = \frac{\sqrt{2}\sqrt{3}}{\sqrt{3}\sqrt{3}} = \frac{\sqrt{6}}{3} \)

d) \( \frac{3}{\sqrt{2-x}} = \frac{3\sqrt{2-x}}{\sqrt{2-x}\sqrt{2-x}} = \frac{3\sqrt{2-x}}{2-x} \)

e) \( \frac{\sqrt{2-x}}{\sqrt{2+x}} = \frac{(2-x)\sqrt{2+x}}{(2-x)(2+x)} = \frac{\sqrt{(2-x)(2+x)}}{2+x} \)

f) \( \frac{6}{3\sqrt{5xy}} = \frac{2}{\sqrt{5xy}} = \frac{2\sqrt{5xy}}{5xy} = \frac{2\cdot\sqrt{5xy}}{5xy} \)

g) \( \frac{\sqrt{xy}}{5\sqrt{pq}} = \frac{\sqrt{xy}\sqrt{pq}}{5\sqrt{pq}\sqrt{pq}} = \frac{\sqrt{pqxy}}{5pq} = \frac{\sqrt{pqxy}}{5pq} \)

h) \( \frac{2\sqrt{xy}}{3\sqrt{3y}} = \frac{2\sqrt{xy}\cdot\sqrt{3y}}{3\sqrt{3y} \sqrt{3y}} = \frac{2\sqrt{3xy^2}}{3\cdot3y} = \frac{2\cdot\sqrt{3y}}{9y} \)

i) \( a = \frac{a}{a} = \frac{\sqrt{a}}{\sqrt{a}} = \frac{a\cdot\sqrt{a}}{\sqrt{a}\cdot\sqrt{a}} = \frac{a\cdot\sqrt{a}}{\sqrt{a}\cdot\sqrt{a}} = \frac{a\cdot\sqrt{a}}{\sqrt{a}} = \sqrt{a} \)

j) \( \sqrt{a} = \sqrt{a} = \frac{a\cdot\sqrt{a}}{\sqrt{a}\cdot\sqrt{a}} = \frac{a\cdot\sqrt{a}}{\sqrt{a}\cdot\sqrt{a}} = \frac{a\cdot\sqrt{a}}{\sqrt{a}} = \sqrt{a} \)
2. Raíz de índice mayor que dos en el denominador

j) \[ \sqrt[3]{2} = \sqrt[3]{2}, \quad \sqrt[3]{\sqrt{3}} = \frac{\sqrt{2} \cdot \sqrt[3]{3}}{3} = \frac{\sqrt{2} \cdot 3^4}{3} = \sqrt[3]{648} \]

k) \[ \frac{1}{\sqrt{x^2 y}} = \frac{1}{\sqrt{x^2 y}}, \quad \frac{\sqrt{x^2 y}}{\sqrt{x^2 y}} = \frac{\sqrt{x^2 y^3}}{\sqrt{x^2 y^3}} = \frac{\sqrt[4]{x^2 y^3}}{\sqrt[4]{x^2 y^3}} = \frac{\sqrt[4]{x^2 y^3}}{\sqrt[4]{x^2 y^3}} = \frac{\sqrt[4]{x^2 y^3}}{\sqrt[4]{x^2 y^3}} = \frac{\sqrt[4]{x^2 y^3}}{xy} \]

l) \[ \frac{6x}{\sqrt[3]{ab^2 x^3}} = \frac{6x}{\sqrt[3]{ab^2 x^3}}, \quad \frac{\sqrt[3]{a^2 b^3 x^2}}{\sqrt[3]{a^2 b^3 x^2}} = \frac{6x \cdot \sqrt[3]{a^2 b^3 x^2}}{ab} = \frac{6x \cdot \sqrt[3]{a^2 b^3 x^2}}{ab} = \frac{6 \cdot \sqrt[3]{a^2 b^3 x^2}}{ab} \]

m) \[ \frac{3x + y}{\sqrt[3]{x - y}} = \frac{3x + y}{\sqrt[3]{x - y}}, \quad \frac{\sqrt[3]{(x - y)^3}}{\sqrt[3]{(x - y)^3}} = \frac{(3x + y) \cdot \sqrt[3]{(x - y)^3}}{(3x + y) \cdot \sqrt[3]{(x - y)^3}} = \frac{(3x + y) \cdot \sqrt[3]{(x - y)^3}}{(3x + y) \cdot \sqrt[3]{(x - y)^3}} = \frac{3 \sqrt[3]{(x - y)^3}}{x - y} \]

n) \[ \frac{3}{\sqrt[3]{x + y}} = \frac{3}{\sqrt[3]{x + y}}, \quad \frac{\sqrt[3]{(x + y)^3}}{\sqrt[3]{(x + y)^3}} = \frac{3 \cdot \sqrt[3]{(x + y)^3}}{3 \cdot \sqrt[3]{(x + y)^3}} = \frac{3 \sqrt[3]{(x + y)^3}}{x + y} \]

o) \[ \frac{3x - 2y}{\sqrt[3]{3x^2 y^4}} = \frac{3x - 2y}{\sqrt[3]{3x^2 y^4}}, \quad \frac{\sqrt[3]{3x^3 y^3}}{\sqrt[3]{3x^3 y^3}} = \frac{(3x - 2y) \cdot \sqrt[3]{3x^3 y^3}}{(3x - 2y) \cdot \sqrt[3]{3x^3 y^3}} = \frac{(3x - 2y) \cdot \sqrt[3]{3x^3 y^3}}{(3x - 2y) \cdot \sqrt[3]{3x^3 y^3}} = \frac{3 \sqrt[3]{3x^3 y^3}}{3xy} \]

p) \[ \frac{a}{\sqrt{a}} = \frac{a}{\sqrt{a}}, \quad \frac{\sqrt{a^2}}{\sqrt{a}} = \frac{\sqrt{a^2}}{\sqrt{a}} = \frac{\sqrt{a^2}}{\sqrt{a}} = \frac{\sqrt{a^2}}{\sqrt{a}} = \frac{\sqrt{a^2}}{\sqrt{a}} = \frac{\sqrt{a^2}}{\sqrt{a}} = \frac{\sqrt{a^2}}{\sqrt{a}} = \frac{\sqrt{a^2}}{\sqrt{a}} = \frac{\sqrt{a^2}}{\sqrt{a}} = \frac{\sqrt{a^2}}{\sqrt{a}} = \frac{\sqrt{a^2}}{\sqrt{a}} = \frac{\sqrt{a^2}}{a} = \frac{\sqrt{a^2}}{a} = \frac{\sqrt{a^2}}{a} \]

q) \[ \frac{a}{\sqrt{a^2}} = \frac{a}{\sqrt{a^2}}, \quad \frac{\sqrt{a^2}}{\sqrt{a}} = \frac{\sqrt{a^2}}{\sqrt{a}} = \frac{\sqrt{a^2}}{\sqrt{a}} = \frac{\sqrt{a^2}}{\sqrt{a}} = \frac{\sqrt{a^2}}{\sqrt{a}} = \frac{\sqrt{a^2}}{\sqrt{a}} = \frac{\sqrt{a^2}}{\sqrt{a}} = \frac{\sqrt{a^2}}{\sqrt{a}} = \frac{\sqrt{a^2}}{\sqrt{a}} = \frac{\sqrt{a^2}}{a} = \frac{\sqrt{a^2}}{a} = \frac{\sqrt{a^2}}{a} \]
3. Racionalizar un binomio con una o dos raíces cuadradas: conjugado

\[ \frac{1}{\sqrt{2} - \sqrt{3}} = \frac{1}{\sqrt{2} - \sqrt{3}} \left( \frac{\sqrt{2} + \sqrt{3}}{\sqrt{2} + \sqrt{3}} \right) = \frac{(\sqrt{2} + \sqrt{3})}{(\sqrt{2} - \sqrt{3}) \cdot (\sqrt{2} + \sqrt{3})} = \frac{(\sqrt{2} + \sqrt{3})}{\left( (\sqrt{2})^2 - (\sqrt{3})^2 \right) / 2} = \frac{(\sqrt{2} + \sqrt{3})}{2 - 3} = \frac{-(\sqrt{2} + \sqrt{3})}{-(\sqrt{2} - \sqrt{3})} = \frac{\sqrt{2}}{3 - \sqrt{5}} = \frac{\sqrt{2}}{3 - \sqrt{5}} \left( \frac{3 + \sqrt{5}}{3 + \sqrt{5}} \right) = \frac{\sqrt{2} \cdot (3 + \sqrt{5})}{(3 - \sqrt{5}) \cdot (3 + \sqrt{5})} = \frac{\sqrt{2} \cdot (3 + \sqrt{5})}{3^2 - (\sqrt{5})^2} = \frac{\sqrt{2} \cdot (3 + \sqrt{5})}{9 - 5} = \frac{\sqrt{2} \cdot (3 + \sqrt{5})}{3 \cdot \sqrt{2} + 2 \cdot \sqrt{5}} = \frac{3 \cdot \sqrt{2} + \sqrt{10}}{4} \]

\[ \frac{\sqrt{3} - 2}{2 + 2\sqrt{3}} = \frac{\sqrt{3} - 2}{2 + 2\sqrt{3}} \left( \frac{2 - 2\sqrt{3}}{2 - 2\sqrt{3}} \right) = \frac{(\sqrt{3} - 2)(2 - 2\sqrt{3})}{(2 + 2\sqrt{3})(2 - 2\sqrt{3})} = \frac{(\sqrt{3} - 2)(2 - 2\sqrt{3})}{(2^2 - (2\sqrt{3})^2)} = \frac{(\sqrt{3} - 2)(2 - 2\sqrt{3})}{4 - 2^2(\sqrt{3})^2} \]

\[ \frac{\sqrt{3} - 2}{2 + 2\sqrt{3}} = \frac{\sqrt{3} - 2}{2 + 2\sqrt{3}} \left( \frac{2 - 2\sqrt{3}}{2 - 2\sqrt{3}} \right) = \frac{(\sqrt{3} - 2)(2 - 2\sqrt{3})}{(2 + 2\sqrt{3})(2 - 2\sqrt{3})} = \frac{(\sqrt{3} - 2)(2 - 2\sqrt{3})}{(2^2 - (2\sqrt{3})^2)} = \frac{(\sqrt{3} - 2)(2 - 2\sqrt{3})}{4 - 2^2(\sqrt{3})^2} \]

\[ \frac{\sqrt{3} - 2}{2 + 2\sqrt{3}} = \frac{\sqrt{3} - 2}{2 + 2\sqrt{3}} \left( \frac{2 - 2\sqrt{3}}{2 - 2\sqrt{3}} \right) = \frac{(\sqrt{3} - 2)(2 - 2\sqrt{3})}{(2 + 2\sqrt{3})(2 - 2\sqrt{3})} = \frac{(\sqrt{3} - 2)(2 - 2\sqrt{3})}{(2^2 - (2\sqrt{3})^2)} = \frac{(\sqrt{3} - 2)(2 - 2\sqrt{3})}{4 - 2^2(\sqrt{3})^2} \]

\[ \frac{\sqrt{x}}{2 - \sqrt{x}} = \frac{\sqrt{x}}{2 - \sqrt{x}} \left( \frac{2 + \sqrt{x}}{2 + \sqrt{x}} \right) = \frac{(\sqrt{x})(2 + \sqrt{x})}{(2 - \sqrt{x})(2 + \sqrt{x})} = \frac{(\sqrt{x})(2 + \sqrt{x})}{2^2 - (\sqrt{x})^2} = \frac{(\sqrt{x})(2 + \sqrt{x})}{4 - x} \]

\[ \frac{(\sqrt{x} + \sqrt{x})}{(4 - x)} = \frac{(\sqrt{x} + \sqrt{x})}{(4 - x)} \left( \frac{2 + \sqrt{x}}{2 + \sqrt{x}} \right) = \frac{(\sqrt{x} + \sqrt{x})(2 + \sqrt{x})}{(4 - x)(2 + \sqrt{x})} = \frac{(\sqrt{x} + \sqrt{x})(2 + \sqrt{x})}{2^2 - (\sqrt{x})^2} = \frac{(\sqrt{x} + \sqrt{x})(2 + \sqrt{x})}{4 - x} \]

\[ \frac{(\sqrt{x} + \sqrt{x})}{(4 - x)} = \frac{(\sqrt{x} + \sqrt{x})}{(4 - x)} \left( \frac{2 + \sqrt{x}}{2 + \sqrt{x}} \right) = \frac{(\sqrt{x} + \sqrt{x})(2 + \sqrt{x})}{(4 - x)(2 + \sqrt{x})} = \frac{(\sqrt{x} + \sqrt{x})(2 + \sqrt{x})}{2^2 - (\sqrt{x})^2} = \frac{(\sqrt{x} + \sqrt{x})(2 + \sqrt{x})}{4 - x} \]
v) 
\[ \frac{2 - \sqrt{3}}{2 + \sqrt{3}} = \frac{(2 - \sqrt{3})}{(2 + \sqrt{3}) \cdot (2 - \sqrt{3})} = \frac{(2 - \sqrt{3})^2}{(2^2 - (\sqrt{3})^2)} = \frac{(2 - \sqrt{3})^2}{(4 - 3)} = \frac{(2 - \sqrt{3})^2}{1} = (2 - \sqrt{3})^2 = 2^2 - 2 \cdot 2 \cdot \sqrt{3} + (\sqrt{3})^2 = 4 - 4 \cdot \sqrt{3} + 3 = 7 - 4 \cdot \sqrt{3} \\
\]

\[ \frac{3\sqrt{2} + 2\sqrt{3}}{3\sqrt{2} - 2\sqrt{3}} = \frac{(3\sqrt{2} + 2\sqrt{3})}{(3\sqrt{2} - 2\sqrt{3}) \cdot (3\sqrt{2} + 2\sqrt{3})} = \frac{(3\sqrt{2} + 2\sqrt{3})^2}{(3\sqrt{2} - 2\sqrt{3}) \cdot (3\sqrt{2} + 2\sqrt{3})} = \frac{(3\sqrt{2} + 2\sqrt{3})^2}{(3\sqrt{2})^2 - (2\sqrt{3})^2} = \]

w) 
\[ \frac{(3\sqrt{2})^2 + 2 \cdot 3 \cdot \sqrt{2} \cdot 2 \sqrt{3} + (2 \sqrt{3})^2}{3^2 \cdot (\sqrt{2})^2 - 2^2 (\sqrt{3})^2} = \frac{9 \cdot 2 + 12 \cdot \sqrt{6} + 4 \cdot 3}{9 \cdot 2 - 4 \cdot 3} = \frac{18 + 12 \cdot \sqrt{6} + 12}{18 - 12} = \frac{30 + 12 \cdot \sqrt{6}}{6} = \frac{6 \cdot (5 + 2 \cdot \sqrt{6})}{6} = 5 + 2 \cdot \sqrt{6} \\
\]

x) 
\[ \frac{2\sqrt{5} - \sqrt{7}}{2\sqrt{5} + \sqrt{7}} = \frac{2\sqrt{5} - \sqrt{7}}{2\sqrt{5} + \sqrt{7}} \cdot \frac{2\sqrt{5} - \sqrt{7}}{2\sqrt{5} - \sqrt{7}} = \frac{(2\sqrt{5} - \sqrt{7})^2}{(2\sqrt{5})^2 - (\sqrt{7})^2} = \frac{(2\sqrt{5})^2 - 2 \cdot 2\sqrt{5} \cdot \sqrt{7} + (\sqrt{7})^2}{(2\sqrt{5})^2 - (\sqrt{7})^2} = \]

\[ \frac{4 \cdot 5 - 4 \cdot \sqrt{35} + 7}{4 \cdot 5 - 7} = \frac{20 - 4 \cdot \sqrt{35} + 7}{20 - 7} = \frac{27 - 4 \sqrt{35}}{13} \]

y) 
\[ \frac{\sqrt{12} - \sqrt{243} + \sqrt{3}}{\sqrt{300} + \sqrt{192}} = \frac{\sqrt{3} \cdot 2^2 - \sqrt{3^5} + \sqrt{3}}{\sqrt{3^2} \cdot 2^2 + \sqrt{2^6} \cdot 3} = \frac{2 \sqrt{3} - 9 \sqrt{3} + \sqrt{3}}{10 \sqrt{3} + \sqrt{8} \sqrt{3}} = \frac{-6 \sqrt{3}}{18 \sqrt{3}} = \frac{-6}{18} = \frac{1}{3} \]